intensity of the secondary vortices close to the surface and the maximum generation of turbulent fluctuations is observed at the boundary of the transitional layer, for $\delta_{\mathrm{OS}}=3 \delta_{0}$ the maximum energy transport by turbulent fluctuations to the surface occurs through the secondary vortices, which leads to an increase of heat transfer. In the range $\delta_{O S} / \delta_{0}>3$ a decrease of the heat transfer must occur with the increase of $\delta_{O S} / \delta_{0}$, i.e., the increase of Reynolds number and decrease of the frequency of oscillation must lead to a decrease of the effect of oscillating flow on heat transfer, which agrees with the available experimental data. However, this assumption requires additional experimental investigation in the range of low-frequency oscillations.

According to the investigations carried out here, the results of the tests on the maximum heat transfer for $\delta_{o s} / \delta_{0} \gtrless 3$ can be generalized by the criterial equation

$$
\begin{equation*}
K_{\max }=1+3,13 \cdot 10^{-3}\left[\frac{\operatorname{Re}_{0}^{1.75}}{\operatorname{Re}_{\omega}}\right]^{0.5}\left[\frac{\Delta(\rho u)_{0}}{(\rho u)_{0}}\right]_{\max } \tag{6}
\end{equation*}
$$

## NOTATION

$d_{0}$, channel diameter; L, channel length; $\rho$, density; $u$, velocity; $P$, pressure; $T$, temperature; $f$, frequency; $\omega$, angular frequency; $\tau_{W}$, tangential stress on channel wall; $\delta_{o s}$, thickness of the oscillating layer; $\delta_{0}$, thickness of the stationary viscous sublayer; $\nu$, kinematic viscosity coefficient; Nu, Nusselt number; $K=N u / N u_{0}$, relative heat-transfer coefficient; $\operatorname{Re}_{0}=u_{0} \mathrm{~d}_{0} / \nu$, Reynolds number; $R e_{\omega}=\omega \mathrm{d}_{0}^{2} / \nu$, oscillating Reynolds number. Indices: 0 , averaged parameters; $\Delta$, fluctuation parameters.

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## INVESTIGATION OF HEAT TRANSFER IN THE ENTRANCE

## REGION OF A FLAT PLATE PARALLEL TO THE FLOW

## WITH A LEADING EDGE IN THE FORM OFA

ONE-SIDED WEDGE
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Relations are given for calculating the local heat transfer in the dynamic initial section of a longitudinally washed generator of a one-sided wedge for the case of constant wall temperature and turbulent boundary layer.

Leading edges with the profile of a sharp one-sided wedge, shown schematically in Fig. 1, are frequently encountered in the natural components of power machinery and in model experiments related to the study of heat transfer and flow over longitudinally washed surfaces $[1,2,13]$. When the angle $\beta$ is quite small and the velocity field in the incident stream is uniform, it is usually assumed that mixed flow is generated in the boundary layer at the edge of a wedge oriented in the direction of the velocity vector $W$ (the edge A in Fig. 1). However, it is known that leading edges with an angle near $90^{\circ}$ cause flow separation $[4,10,11]$.

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Fig. 1. Geometric characteristics of the leading edges (Nos. 1-4 are plates with imbedded heat-flux sensors, and Nos. 5-8 are plates with imbedded alpha-calorimeters): 1) water-cooled plate; 2) water-cooled entrance adapter; 3) heat-flux sensor; 4) textolite plate; 5) alpha-calorimeter.

The objective of the present paper is to try to establish a relationship between the angle of a wedgeshaped leading edge and the local heat-transfer conditions in the dynamic entrance section of a longitudinally washed flat plate. Two groups of experiments were performed. In the first group a water-cooled steel plate of thickness $\delta=8.0 \mathrm{~mm}$ with imbedded, insulated heat-flux sensors was used. The plate was described in [8, 9]. The plate was equipped, in addition, with three interchangeable water-cooled entrance sections, whose structure is illustrated in Fig. 1. The wedge angles $\beta$ for the three adapters were as follows: No. $1-90^{\circ}$; No. $2-70^{\circ}$; No. $3-40^{\circ}$; and No. $4-20^{\circ}$. Without the adapter the entrance section had $\beta=90^{\circ}$, No. 1 。 Special features of the technique for measurement of local heat-transfer coefficients using the heat-flux sensors were examined in $[8,9,12]$.

In the second group of experiments heat-transfer measurements in the entrance section were made using an unsteady method and imbedded alpha-calorimeters $[3,6,7]$. Here the plate thickness was varied, $\delta=2.0$; $5.0 ; 7.8 ; 12.0 \mathrm{~mm}$, for a constant value of the angle equal to $90^{\circ}$ (entrance sections Nos. 5-8 in Fig. 1).

The experiments were conducted in a closed wind tunnel with working-section cross section of $280 \times 280$ $\mathrm{mm}^{2}[5,6]$ at temperatures, velocities, and air-stream turbulence levels of $\mathrm{T}_{\mathrm{flow}}=50-55^{\circ} \mathrm{C}, \mathrm{W}=5-8 \mathrm{~m} / \mathrm{sec}$, $\varepsilon=2.2 \pm 0.2 \%$, and with constant temperature along the heat-generating surface. In the first group of tests $\mathrm{T}_{\mathrm{flow}} / \mathrm{T}_{\mathrm{W}}=1.1$ and in the second group, $\mathrm{T}_{\mathrm{flow}} / \mathrm{T}_{\mathrm{W}} \approx 1.0$ 。 The plate was located on the longitudinal axis of the wind tunnel.

Figure 2a shows the results of measurements of local heat transfer on a plate with $\delta=8.0 \mathrm{~mm}$, heated along the entire length, with flow velocity $W=11.0 \mathrm{~m} / \mathrm{sec}$, and with different entrance section wedge angles. The test points were taken for the region $X<120 \mathrm{~mm}$, since for $X>150 \mathrm{~mm}$ the local heat-transfer coefficients were practically independent of the entrance conditions [7,9]. The dashed line is calculated using Eq. (1), described in [7] for the hypothetical case where a turbulent boundary-layer flow begins at the beginning of the plate

$$
\begin{equation*}
S t=0.0146 \mathrm{Re}_{X}^{-0,16+3.25 \mathrm{Re}_{X}^{-0.53}} \operatorname{Pr}^{-0.6} \tag{1}
\end{equation*}
$$

The relative location of the curves in Fig. 2a allows one to comment on the following laws which are stable and repetitive over the entire velocity range investigated. For $\beta \leq 70^{\circ}$ and $X \geq 15 \mathrm{~mm}$ there is no sign of mixed flow in the shape of the local heat-transfer-coefficient distributions. The relations $\alpha_{l o c}=f(X)$ are monotonic, and their slopes increase with increase of $\beta$. When the angle is $90^{\circ}$, the curve $\alpha_{\text {loc }}=\mathrm{f}(\mathrm{X})$ acquires a shape typical of separated flow [10,11]. The fall in heat-transfer intensity immediately adjacent to the entrance is replaced here by a sharp rise in the values of $\alpha$ loc, and the parameter passes through a maximum at


Fig. 2. Results of measurements of local heat-transfer coefficients in the dynamic entrance section of a plate with $\delta=$ 8.0 mm for $W=11.0 \mathrm{~m} / \mathrm{sec}$ and total heating; a) distribution $\left.\alpha_{l o c}=f(X): 1\right)$ No. 1 entrance; 2) No. 2 ; 3) No. 3; 4) No. 4 ; 5) using Eq. (1); the numbering of the leading edges is the same as in Fig. 1; b) the influence of the angle $\beta$ on the local heat-transfer intensity at fixed points in the entrance section: $\alpha$ loc, $\mathrm{W} / \mathrm{m}^{2}$ 。deg.
a distance from the leading edge roughly equal to two plate thicknesses. As was shown in $[4,7,9]$, there is flow reattachment at the maximum heat-transfer point. To the right of this point the local heat-transfer coefficients decrease smoothly, but remain high in comparis on with leading edges Nos. 2-4 over the whole extent of the initial section. It should be noted that the position of maximum heat transfer is gradually displaced upstream as the velocity increases.

As can be seen from Fig. 2 b , which shows the interpolated graphs of $\alpha_{\mathrm{loc}}=\mathrm{f}(\beta)$ for $\mathrm{X}=20 \mathrm{~mm}$. and $\mathrm{X}=$ 40 mm , the local heat-transfer intensity at fixed points on the entrance section to the right of the maximum depends almost linearly on the angle $\beta$, and the test data agree quite satisfactorily with calculations using Eq. (1), if we take into account that the dashed line in Fig. 2a corresponds to the case $\beta=0^{\circ}$.

Judging from the results of a comparative analysis of the first group of tests, a computational scheme for heat transfer in the entrance section in the range $L_{d y n}>X>X_{\max }$ can be based on a relationship of the type

$$
\begin{equation*}
\mathrm{St}_{\mathrm{dyn}}=\xi \mathrm{St} \tag{2}
\end{equation*}
$$

where St is the Stanton number, computed from Eq. (1), and $\xi$ is a correction reflecting the influence of geometric and dynamical factors specific to the entrance section.

The main problem in tests of the second group was a qualitative analysis of the flow and heat-transfer regimes in the entrance section. The experimental data indicate that the heat-transfer-coefficient distributions on plates of different thicknesses with $\beta=90^{\circ}$ are similar, as a function of $X$, to the curve of Fig. 2a, obtained for entrance section No. 1.* Omly for entrance section No. 5 was there no clearly marked displacement of the heat-transfer maximum, since the size of the alpha-calorimeters was greater than the quantity $\mathrm{X}_{\mathrm{max}}$.

Figure 3 shows the dimensionless coordinates of the heat-transfer maxima $X_{m a x} / \delta$, determined from the results of measurements in the two groups of tests and reduced as a function of Reynolds number based on plate thickness. The test points show satisfactory agreement and fall along a curve which tends asymptotically to a limiting value $\operatorname{Re} \delta>5 \cdot 10^{3}$ equal to 2 for $\mathrm{X}_{\max } / \delta$ 。

The nature of the experimental curve in Fig. 3, together with the data of Fig. 2, supports the assumption that for any wedge angles $\beta$, including very small values, and plate thicknesses $\delta$, the sharp leading edge remains a source of turbulent perturbations or for generating separation in the entrance section (at least for a turbulence level in excess of $2.0 \%$ in the oncoming stream). Similar statements were made also in [1,2,13], where it was observed also that a turbulent boundary layer appeared practically at the beginning of the flow, on the longitudinally washed surface of a one-sided wedge, in spite of the conventional viewpoint. The flow in the boundary layer becomes mixed only in the cases where there is transverse suction of air through the leading edge, or where it is rounded with radius R , given by the relation $\mathrm{WR} / \nu<500$ [3].

If we use the dimensionless angle $\bar{\beta}=\beta / 90^{\circ}$ and the coordinate $\bar{X}=X / \delta+4$, then under complete heating conditions the computational relation for the correction $\xi$ can be written as follows:

[^0]

Fig. 3. Results of determining the maximum heattransfer point in the entrance section of these plates for $\beta=90^{\circ}$ : 1) entrance No. 1; 2) entrance No. 6; 3) entrance No. 7; 4) entrance No. 8. The leading edge numbering is the same as in Fig. 1 。


Fig. 4. Results of dimensionless reduction of the test data on local heat transfer in the entrance section of the plate for $\mathrm{L}_{\mathrm{dyn}}>\mathrm{X}>\mathrm{X}_{\max }$, various values of the leadingedge wedge angle $\beta$, and with heating over the entire surface.

$$
\begin{equation*}
\xi=1 \div \bar{\beta}\left(1-1.2 \cdot 10^{-5} \mathrm{Re}_{\delta} \cdot \overline{\beta^{2}}\right) \exp \left(1-0.014 \bar{X}^{2}\right) . \tag{3}
\end{equation*}
$$

Figure 4 shows the results of measurements of local heat-transfer coefficients in the entrance section of a longitudinally washed generator of a one-sided wedge 1-4 (entrance sections Nos. 1-4), represented in the form of the relation $S t / \xi=f\left(\operatorname{Re}_{\mathrm{X}}\right)$. Curve I, calculated from Eq. (1), can be fitted to the test data with a scatter not exceeding $15-18 \%$. Equation (3) has been confirmed experimentally in the following range: $\overline{\mathrm{X}}=6$ $30 ; \operatorname{Re}_{\delta}=5 \cdot 10^{2}-3 \cdot 10^{4} ; \bar{\beta}=0.2-1.0 ; \operatorname{Re}_{\mathrm{X}}=10^{3}-5 \cdot 10^{5}$ 。

## NOTATION

$\delta$, plate thickness, $m ; X$, coordinate calculated from the leading edge, $m ; X_{m a x}$, coordinate of the maximum heat-transfer coefficient, $\mathrm{m} ; \overline{\mathrm{X}}=\mathrm{X} / \delta+4$, dimensionless coordinate; $\mathrm{I}_{\mathrm{d} y \mathrm{n}}$, extent of the dynamic entrance section, $m ; R$, radius of curvature of the leading edge, $m ; \beta$, angle of the leading-edge wedge, deg; $\overline{3}=\beta / 90^{\circ}$, dimensionless leading-edge wedge angle; $\alpha$ loc, local heat-transfer coefficient, $\mathrm{W} / \mathrm{m}^{2} \cdot \mathrm{deg} ; \varepsilon$, turbulence level of the air stream, $\%$; $\mathrm{T}_{\text {flow, }}$, air stream temperature, ${ }^{\circ} \mathrm{C}$ or ${ }^{\circ} \mathrm{K} ; \mathrm{T}_{\mathrm{W}}$, wall temperature, ${ }^{\circ} \mathrm{C}$ or ${ }^{\circ} \mathrm{K} ; \xi$, correction to the heat transfer in the entrance section; $\nu$, coefficient of kinematic viscosity of air, $\mathrm{m}^{2} / \mathrm{sec} ; \mathrm{St}_{\mathrm{dyn}}$, St, Stanton numbers for the dynamic entrance section and for the region with developed flow turbulence in the boundary layer; $\mathrm{Re}_{\mathrm{X}}$, Reynolds number, based on the heat-plate length; Re $\mathrm{e}_{\delta}$, Reynolds number based on the plate thickness.

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## HEAT EXCHANGE IN LAMINAR FLOW THROUGH

## PLANAR AND QUASIPLANAR CHANNELS OF VARIABLE

## CROSS SECTION

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Formulas are derived for both the local and averaged over the surface Nusselt number. Experimental results are presented and the limits of applicability of the calculated formulas are indicated.

We denote channels whose walls lie in parallel planes as planar, while channels with a constant distance between the walls and with a radius of curvature of the channel wall generatrix much greater than this distance will be referred to as quasiplanar. The analytic solution of the problem of heat exchange in the laminar flow of a liquid through plane channels has been considered in many studies (for example, [1]). However, the available analytic studies are dedicated to heat exchange in planar channels of constant cross section, in which the mean motion velocity is constant. Experimental data for planar channels of variable cross section were presented in [2,3], but these are of a partial character.

We will consider the analytic solution of the problem of heat exchange for a laminar liquid flow in quasiplanar channels (Fig. 1a, c). A schematic diagram of a portion of the channel is shown in Fig. 1d. In formulating the problem we make the following assumptions, in addition to those usually made ([1], p. 76): a) the liquid flow and temperature field have azimuthal symmetry; b) the channel width is a fixed function of the longitudinal coordinate (by channel width we understand the length of the line perpendicular to the flow lines and located in a plane parallel to the channel walls); c) the velocity profile over the channel height at section x is parabolic:

$$
\begin{equation*}
\omega_{x}=\frac{3}{2} \bar{\omega}\left(1-\frac{y^{2}}{h^{2}}\right)=\frac{3}{2} \frac{G_{m}}{\delta f(x) \rho}\left(1-\frac{y^{2}}{h^{2}}\right) . \tag{1}
\end{equation*}
$$

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[^0]:    *Some of these test data were published in $[3,6]$.

